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OPTIMAL RECURSIVE MAXIMUM LIKELIHOOD ESTIMATION(U)

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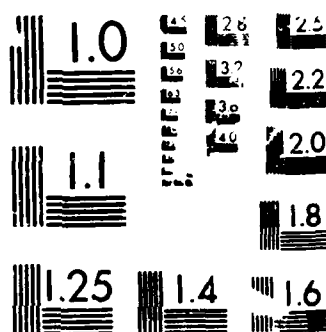
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## OPTIMAL RECURSIVE MAXIMUM LIKELIHOOD ESTIMATION

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Department of Electrical Engineering, Carnegie Mellon University, Pittsburgh, PA.**Abstract:** In this paper we derive stochastic differential equations for recursive maximum-likelihood estimates for the joint filtering-parameter estimation problem.**Keywords:** Maximum likelihood estimate; stochastic differential equation; Hamilton-Jacobi Equation; Nonlinear Filtering

## 1. INTRODUCTION

In this paper we would like to consider the joint states and parameter estimation problem for the following non-linear stochastic differential system:

$$dx(t) = f(x(t), \theta)dt + g(x(t), \theta)dw(t), \quad 0 \leq t \leq T \quad (1.1)$$

with the observation system

$$dy(t) = h(x(t), \theta)dt + dv(t), \quad 0 \leq t \leq T. \quad (1.2)$$

In the above,  $w(t)$  and  $v(t)$  are standard independent Brownian motions,  $f, g, h$  are at least thrice continuously differentiable with bounded derivatives with respect to  $x \in R$  and  $\theta \in R$  and  $g(x, \theta) \neq 0, \forall x, \theta \in R$ .

In addition we assume

$$E \int_0^T |h(x(t), \theta)|^2 dt < \infty, \quad (1.3)$$

and the initial state satisfies

$$\text{Either i) } x(0) = x_0 \in R \quad (1.4)$$

$$\text{ii) } x(0) = x_0, \text{ a random variable with density } p_0(x) \in C_b^2(R; R), \quad p_0(x) > 0.$$

Let  $\phi_{s,t}(x)$  denote the solution of the stochastic differential equation (1.1) starting at  $x_s = x$ . Then from a result of Kunita [2], we know that  $\phi_{s,t}^{-1}$  is a  $C^1$ -diffeomorphism, and the inverse map  $\phi_{s,t}^{-1}$  satisfies a backward stochastic differential equation.

Let

$$\begin{aligned} A(\theta, t) &= \exp\left(\int_0^t h(x(s), \theta) dy(s) - \frac{1}{2} \int_0^t h^2(x(s), \theta) ds\right) \\ &= \exp\left[\int_0^t \tilde{h}(\phi_{s,t}^{-1}(x(t)), \theta) dy(s) - \frac{1}{2} \int_0^t \tilde{h}(\phi_{s,t}^{-1}(x(t)), \theta)^2 ds\right], \end{aligned} \quad (1.5)$$

where  $\sim$  denotes a backway stochastic differential (and backward Ito integral respectively).

Let

$$L(\theta, t) = E(A(\theta, t) | x(t) = z), \quad (1.6)$$

where  $E$  denotes expectation with respect to the path space measure of  $x(\cdot)$ .

As a criterion, we choose as an estimate

$$\hat{z}(t) = \underset{z, \theta}{\text{Arg Max}} L(z, \theta, t).$$

which is a maximum likelihood criterion.

2. STOCHASTIC HAMILTON-JACOBI BELLMAN EQUATION FOR  $L(z, \theta, t)$ 

Using the work of Fleming-Mitter [1] and the theory of backway stochastic differential equations [cf. Kunita, loc.cit] one can show that

$$S(z, \theta, t) = -\ln L(z, \theta, t) \quad (2.1)$$

satisfies the stochastic Bellman Hamilton-Jacobi equation:

$$\begin{aligned} dS(x, \theta, t) &= \sigma(x, \theta) (S_{xx} - S_x^2) dt + \alpha(x, \theta) S_x dt \\ &\quad + h(x, \theta)^2 dt - h(x, \theta) dy(t) \end{aligned} \quad (2.2)$$

where

$$\begin{aligned} \sigma(x, \theta) &= \frac{1}{2} g(x, \theta)^2 \\ \alpha(x, \theta) &= 2\sigma_x - 2\sigma(x, \theta) \bar{S}_x - f(x, \theta) \\ \bar{S} &= -\ln p^\theta(x, t) \end{aligned}$$

where under our assumptions  $p^\theta(x, t)$ , the density corresponding to the  $x(\cdot)$  process exists and is positive for all  $x, t$ .

We now define a recursive maximum likelihood estimate. By applying the Generalized Ito Differential Rule [cf. Kunita, loc.cit], we get

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$$\partial_t VS + V^2 S d\xi(t) + \frac{1}{2} V^3 S d\langle \xi, \xi \rangle_t - V(Vh) d\langle y, \xi \rangle_t = 0 \quad (2.3)$$

where

$$V = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial \theta} \end{pmatrix}, \quad \xi(t) = \begin{pmatrix} \xi(t) \\ \theta(t) \end{pmatrix}$$

which is obtained from the stationarity condition

$$VS = 0. \quad (2.4)$$

In (2.3) all partial derivatives are computed along  $(\xi, \theta)$  which is obtained from the stationarity condition (2.4).

Assuming  $V^2 S$  is invertible, we obtain a maximum likelihood trajectory for  $\xi(t)$  from (2.2), (2.3) and (2.4) and using  $\partial_t VS = VdS$ .

A rigorous derivation of these results will be presented elsewhere.

#### REFERENCES

Fleming, W.H. and S.K. Mitter: Optimal Control and Nonlinear Filtering for Nondegenerate Diffusion Processes, Stochastics, 8, 1982, pp. 63-77.

Kunita, H.: Stochastic Differential Equations and Stochastic Flows of Diffeomorphism, Ecole de Probabilites de Saint-Flour XII - 1982.

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